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Anomalous dimensions of Wilson operators in $N = 4$ SYM theory

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Abstract

We present the results of two-loop calculations of the anomalous dimension matrix for the Wilson twist-2 operators in the $N = 4$ Supersymmetric Yang–Mills theory for polarized and unpolarized cases. This matrix can be transformed to a triangle form by the same similarity transformation as in the leading order. The eigenvalues of the anomalous dimension matrix are expressed in terms of an universal function with its argument shifted by integer numbers. In the end we discuss relations between the weak and strong coupling regimes in the framework of the AdS/CFT correspondence.

Parton distributions in QCD satisfy the Balitsky–Fadin–Kuraev–Lipatov (BFKL) [1] and Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) [2,3] equations. Next-to-leading corrections to the BFKL equation were calculated only recently [4]. It is natural to generalize these equations to the supersymmetric case (see Refs. [5–7] and references therein). Indeed, the supersymmetric field theories have a number of amazing properties, such as a cancellation of quadratic divergencies and non-renormalization theorems for interaction terms in the Lagrangian. Moreover, the supersymmetry is an excellent technical playground for QCD. For example, the empirically established Dokshitzer relation [3] among elements of the leading order anomalous dimension matrix in the $N = 1$ supersymmetric limit provides a non-trivial check of results of higher order calculations. Another interesting example is the relation between the BFKL and DGLAP equations in the $N = 4$ Supersymmetric Yang–Mills (SYM) theory [5,6]. In this model one can obtain anomalous dimensions of the multiplicatively renormalizable twist-2 operators from the eigenvalues of the BFKL kernel [7]. These operators are certain linear combinations of the Wilson operators appearing in the theoretical description of the deep-inelastic ep scattering [5,6] (note, that in the $N = 4$ SYM the beta function is zero and the Bjorken scaling for structure functions is strongly violated). Using some assumptions the authors of Ref. [6] derived also an expression for the universal anomalous dimension for the $N = 4$ model in the two-loop approximation. Moreover, the eigenvalues of the anomalous dimension matrices in the polarized and unpolarized cases were obtained from this universal anomalous dimension by an appropriate integer shift of its argument. In this Letter we present the results of direct two-loop calculations of these matrices in the $N = 4$ SYM theory.

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Now the anomalous dimensions of twist-2 operators in QCD are known up to two loops both for the unpolarized [8,9] and polarized [10,11] cases. In the $N=4$ SYM theory [12] there are one gluon g, four Majorana fermions q, three scalars and three pseudoscalars which can be unified in three complex scalars φ. All particles belong to the adjoint representation of the gauge group $SU(N_c)$. The transition from QCD to the $N=4$ SYM theory can be performed if one puts in the final expressions $C_A=C_F=N_c$, $T_f=2N_c$ (the last substitution follows from the fact, that each gluino q_i from four Majorana particles gives a half of the Dirac spinor contribution). Furthermore, one should take into account the diagrams with virtual scalars in the polarized structure functions and the graphs with external scalars in the non-polarized distributions. In the last case the anomalous dimension matrix extends to 3×3. Below we calculate the anomalous dimensions of the following gauge-invariant twist-2 operators:

$$\mathcal{O}^{g}_{\mu_1,\ldots,\mu_j}=\widehat{S}G_{\rho\mu_1}D_{\mu_2}D_{\mu_3}\cdots D_{\mu_{j-1}}G_{\rho\mu_j}, \tag{1}$$

$$\widetilde{\mathcal{O}}^{g}_{\mu_1,\ldots,\mu_j}=\widehat{S}G_{\rho\mu_1}D_{\mu_2}D_{\mu_3}\cdots D_{\mu_{j-1}}\widetilde{G}_{\rho\mu_j}, \tag{2}$$

$$\mathcal{O}^{q}_{\mu_1,\ldots,\mu_j}=\widehat{S}\bar{\Psi}\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_j}\Psi, \tag{3}$$

$$\widetilde{\mathcal{O}}^{q}_{\mu_1,\ldots,\mu_j}=\widehat{S}\bar{\Psi}\gamma_5\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_j}\Psi, \tag{4}$$

$$\mathcal{O}^{\varphi}_{\mu_1,\ldots,\mu_j}=\widehat{S}\bar{\Phi}D_{\mu_1}D_{\mu_2}\cdots D_{\mu_j}\Phi, \tag{5}$$

where D_μ are covariant derivatives; the spinor Ψ and field tensor $G_{\rho\mu}$ describe gluinos and gluons, respectively, and Φ is the complex scalar field appearing in the $N=4$ supersymmetric model. The symbol $\widehat{S}$ implies a symmetrization of the tensor in the Lorenz indices $\mu_1,\ldots,\mu_j$ and a subtraction of its traces. The anomalous dimension matrices can be written as follows for the unpolarized

$$\gamma_{\text{unpol}}=\begin{vmatrix}\gamma_{gg} & \gamma_{gq} & \gamma_{g\varphi}\\ \gamma_{qg} & \gamma_{qq} & \gamma_{q\varphi}\\ \gamma_{\varphi g} & \gamma_{\varphi q} & \gamma_{\varphi\varphi}\end{vmatrix} \tag{6}$$

and polarized cases

$$\gamma_{\text{pol}}=\begin{vmatrix}\tilde{\gamma}_{gg} & \tilde{\gamma}_{gq}\\ \tilde{\gamma}_{qg} & \tilde{\gamma}_{qq}\end{vmatrix}. \tag{7}$$

Note, that in the supermultiplet of twist-2 operators there are also operators with fermion quantum numbers and operators anti-symmetric in two Lorentz indices [13].

Our approach is similar to that of Refs. [9,10]. In particular we calculated unrenormalized matrix elements of the partonic operators sandwiched between the scalars, fermion and gluon states[1] and the anomalous dimensions were extracted from the expansion of the matrix elements through the renormalization group coefficients, with the condition, that the renormalized matrix elements satisfy the Callan–Symanzik equations. In our calculations we used the modified minimal subtraction scheme ($\overline{\text{MS}}$). Because this scheme violates the supersymmetry, the results were transformed to the dimensional reduction scheme ($\overline{\text{DR}}$) [18], explicitly preserving supersymmetry at least in the two-loop level. For this purpose we used the same procedure as in Ref. [19]. Namely, the difference of two-loop results in $\overline{\text{MS}}$- and $\overline{\text{DR}}$-schemes was related to the difference of the finite contributions of the corresponding one-loop results.

In the polarized case one needs an appropriate choice for the γ_5-prescription. Our procedure is analogous to that of Ref. [10], which based on "reading point" method [20]. To begin with, in each trace of the γ-matrix product, we pushed γ_5 to the right-hand side using the property of the trace cyclicity. After that we simplified in a straightforward way the product of γ-matrices leaving the γ_5-matrix untouched and used the relation $\mathrm{Tr}\,\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5=-4i\epsilon_{\mu\nu\rho\sigma}$. Then the integration over the loop momenta in the spacetime dimension $D=4-2\varepsilon$

[1] For the calculations we used the program DIANA [14], which calls QGRAF [15] for the generation of Feynman diagrams, and the package MINCER [16] for FORM [17] for the evaluation of two-loop diagrams.

was performed and the contraction between two Levi-Civita tensors (second one appearing from the projector) in four dimensions was done. In the end we introduced an additional renormalization constant to restore the anticommutativity of γ_5 with other γ-matrices in an accordance with Ref. [21].

The final two-loop result for the elements of the anomalous dimension matrix in $N = 4$ SYM theory in the $\overline{\mathrm{DR}}$-scheme has the following form (multiplied by $\alpha_s^2 N_c^2/(4\pi)^2$) in the unpolarized case (for even j)

$$\begin{aligned}
\gamma_{gg}^{(1)}(j) &= \frac{-500}{9(j-1)} - \frac{16}{j^3} + \frac{72}{j^2} + \frac{140}{3j} + \frac{24}{(j+1)^2} - \frac{236}{3(j+1)} - \frac{16}{(j+2)^3} + \frac{176}{3(j+2)^2} + \frac{788}{9(j+2)} \\
&\quad - 16K(j-1) + 16K(j) - 16K(j+1) + 16K(j+2) + \widehat{Q}(j), \\
\gamma_{gq}^{(1)}(j) &= \frac{-500}{9(j-1)} - \frac{16}{j^3} + \frac{72}{j^2} + \frac{140}{3j} - \frac{22}{3(j+1)} + \frac{32}{3(j+2)^2} + \frac{152}{9(j+2)} \\
&\quad - 16K(j-1) + 16K(j) - 8K(j+1), \\
\gamma_{g\varphi}^{(1)}(j) &= \frac{-500}{9(j-1)} - \frac{16}{j^3} + \frac{72}{j^2} + \frac{140}{3j} - \frac{8}{(j+1)^2} + \frac{16}{j+1} - \frac{16}{3(j+2)^2} - \frac{64}{9(j+2)} \\
&\quad - 16K(j-1) + 16K(j), \\
\gamma_{qg}^{(1)}(j) &= \frac{320}{9(j-1)} + \frac{32}{j^3} - \frac{96}{j^2} + \frac{8}{3j} - \frac{96}{(j+1)^2} + \frac{944}{3(j+1)} + \frac{64}{(j+2)^3} - \frac{704}{3(j+2)^2} - \frac{3152}{9(j+2)} \\
&\quad - 32K(j) + 64K(j+1) - 64K(j+2), \\
\gamma_{qq}^{(1)}(j) &= \frac{320}{9(j-1)} + \frac{32}{j^3} - \frac{96}{j^2} + \frac{8}{3j} + \frac{88}{3(j+1)} - \frac{128}{3(j+2)^2} - \frac{608}{9(j+2)} \\
&\quad - 32K(j) + 32K(j+1) + \widehat{Q}(j), \\
\gamma_{q\varphi}^{(1)}(j) &= \frac{320}{9(j-1)} + \frac{32}{j^3} - \frac{96}{j^2} + \frac{8}{3j} + \frac{32}{(j+1)^2} - \frac{64}{j+1} + \frac{64}{3(j+2)^2} + \frac{256}{9(j+2)} - 32K(j), \\
\gamma_{\varphi g}^{(1)}(j) &= \frac{64}{3(j-1)} + \frac{24}{j^2} - \frac{48}{j} + \frac{72}{(j+1)^2} - \frac{236}{j+1} - \frac{48}{(j+2)^3} + \frac{176}{(j+2)^2} + \frac{788}{3(j+2)} \\
&\quad - 48K(j+1) + 48K(j+2), \\
\gamma_{\varphi q}^{(1)}(j) &= \frac{64}{3(j-1)} + \frac{24}{j^2} - \frac{48}{j} - \frac{22}{j+1} + \frac{32}{(j+2)^2} + \frac{152}{3(j+2)} - 24K(j+1), \\
\gamma_{\varphi\varphi}^{(1)}(j) &= \frac{64}{3(j-1)} + \frac{24}{j^2} - \frac{48}{j} - \frac{24}{(j+1)^2} + \frac{48}{j+1} - \frac{16}{(j+2)^2} - \frac{64}{3(j+2)} + \widehat{Q}(j)
\end{aligned}$$

and in the polarized case (for odd j)

$$\begin{aligned}
\tilde{\gamma}_{gg}^{(1)}(j) &= \frac{32}{j^2} - \frac{280}{3j} - \frac{32}{(j+1)^3} + \frac{64}{(j+1)^2} + \frac{280}{3(j+1)} - 32K(j) + 32K(j+1) + \widehat{Q}(j), \\
\tilde{\gamma}_{gq}^{(1)}(j) &= \frac{16}{j^2} - \frac{140}{3j} - \frac{8}{(j+1)^3} + \frac{32}{(j+1)^2} + \frac{142}{3(j+1)} - 16K(j) + 8K(j+1), \\
\tilde{\gamma}_{qg}^{(1)}(j) &= \frac{-64}{j^2} + \frac{568}{3j} + \frac{64}{(j+1)^3} - \frac{128}{(j+1)^2} - \frac{560}{3(j+1)} + 32K(j) - 64K(j+1), \\
\tilde{\gamma}_{qq}^{(1)}(j) &= \frac{-32}{j^2} + \frac{284}{3j} + \frac{16}{(j+1)^3} - \frac{64}{(j+1)^2} - \frac{284}{3(j+1)} + 16K(j) - 16K(j+1) + \widehat{Q}(j),
\end{aligned}$$

where

$$\widehat{Q}(j) = -\frac{4}{3}S_1(j) + 16S_1(j)S_2(j) + 8S_3(j) - 8\widetilde{S}_3(j) + 16\widetilde{S}_{1,2}(j), \tag{8}$$

$$K(j)=\frac{1}{j}\left(\frac{S_1(j)}{j}+S_2(j)+\widetilde{S}_2(j)\right), \tag{9}$$

$$S_k(j)=\sum_{i=1}^{j}\frac{1}{i^k}, \qquad \widetilde{S}_k(j)=\sum_{i=1}^{j}\frac{(-1)^i}{i^k}, \qquad \widetilde{S}_{k,l}(j)=\sum_{i=1}^{j}\frac{1}{i^k}\widetilde{S}_l(i). \tag{10}$$

The analytical continuation of functions $\gamma_{ab}^{(1)}(j)$ $(a,b=g,q,\varphi)$ and $\tilde{\gamma}_{ab}^{(1)}(j)$ $(a,b=g,q)$ to the complex values of j can be done analogously to Refs. [6,22]. The procedure of the analytic continuation together with a detailed description of our method of calculations will be presented elsewhere.

The eigenvalues of the anomalous dimension matrices are given below

$$\gamma_I^{(1)}(j)=\gamma_+^{(1)}(j)=\widehat{Q}(j-2), \tag{11}$$
$$\gamma_{II}^{(1)}(j)=\gamma_0^{(1)}(j)=\widehat{Q}(j), \tag{12}$$
$$\gamma_{III}^{(1)}(j)=\gamma_-^{(1)}(j)=\widehat{Q}(j+2), \tag{13}$$
$$\gamma_{IV}^{(1)}(j)=\tilde{\gamma}_+^{(1)}(j)=\widehat{Q}(j-1), \tag{14}$$
$$\gamma_{V}^{(1)}(j)=\tilde{\gamma}_-^{(1)}(j)=\widehat{Q}(j+1). \tag{15}$$

In fact they coincide with the expressions predicted in Ref. [6]. Indeed, using the two-loop result

$$\begin{aligned}\gamma_+(j)&=\tilde{\gamma}_+(j-1)=\gamma_0(j-2)=\tilde{\gamma}_-(j-3)=\gamma_-(j-4)=\gamma(j)\\&=-\frac{\alpha_s N_c}{\pi}S_1(j-2)+\left(\frac{\alpha_s N_c}{4\pi}\right)^2\widehat{Q}(j-2)\end{aligned} \tag{16}$$

for the universal anomalous dimension $\gamma(j)$ we can redefine $\alpha_s \to \alpha_s(1-\alpha_s N_c/(12\pi))$ to remove in $\widehat{Q}(j)$ the term proportional to $S_1(j)$. After this substitution the above universal function $\widehat{Q}(j)$ in two loops coincides with $16Q(j)$ from Ref. [6].

For the polarized case the Dokshitzer relation is similar to original one (below $\gamma_{ab}^{(1)}(j)=\gamma_{ab}$ and $\tilde{\gamma}_{ab}^{(1)}(j)=\tilde{\gamma}_{ab}$)

$$\tilde{\gamma}_{gg}+\frac{1}{2}\tilde{\gamma}_{qg}=\tilde{\gamma}_{qq}+2\tilde{\gamma}_{gq} \tag{17}$$

and we can find that

$$\tilde{\gamma}_{gg}+\frac{1}{2}\tilde{\gamma}_{qg}=\widehat{Q}(j-1), \tag{18}$$
$$\tilde{\gamma}_{gg}-2\tilde{\gamma}_{gq}=\widehat{Q}(j+1). \tag{19}$$

There are three relations for the unpolarized case

$$\gamma_{gg}+\gamma_{qg}+\gamma_{\varphi g}=\gamma_{gq}+\gamma_{qq}+\gamma_{\varphi q}=\gamma_{g\varphi}+\gamma_{q\varphi}+\gamma_{\varphi\varphi}, \tag{20}$$
$$\gamma_{gg}-4\gamma_{gq}+3\gamma_{g\varphi}=-\frac{\gamma_{qg}}{4}+\gamma_{qq}-\frac{3\gamma_{q\varphi}}{4}=\frac{\gamma_{\varphi g}}{3}-\frac{4\gamma_{\varphi q}}{3}+\gamma_{\varphi\varphi}, \tag{21}$$
$$12\gamma_{gq}-12\gamma_{g\varphi}+3\gamma_{qg}-3\gamma_{q\varphi}+4\gamma_{\varphi g}-4\gamma_{\varphi q}=0 \tag{22}$$

and one can verify that

$$\gamma_{gg}+\gamma_{qg}+\gamma_{\varphi g}=\widehat{Q}(j-2), \tag{23}$$
$$\gamma_{gg}-4\gamma_{gq}+3\gamma_{g\varphi}=\widehat{Q}(j), \tag{24}$$
$$\gamma_{gg}-\gamma_{gq}-\frac{\gamma_{\varphi g}}{3}+\frac{\gamma_{\varphi q}}{3}=\widehat{Q}(j+2). \tag{25}$$

A complete diagonalization of the above anomalous dimension matrices corresponds to the use of a slightly modified basis of the multiplicatively renormalized twist-2 operators in comparison with the leading order [5,6] due to the breakdown of the superconformal invariance (cf. [23]). But Eqs. (17)–(25) are correct in all orders with the replacement of $\widehat{Q}(j-2)$ by the exact universal anomalous dimension $\gamma(j)$. Following the analysis in Ref. [6] the α_s^3 correction to the universal anomalous dimensions $\gamma(j)$ will be constructed as soon as the corresponding QCD anomalous dimensions will be calculated (see Ref. [24]).

Recently there was a great progress in the investigation of the $N=4$ SYM theory in a framework of the AdS/CFT correspondence [25] where the strong-coupling limit $\alpha_s N_c \to \infty$ is described by a classical supergravity in the anti-de Sitter space $AdS_5 \times S^5$. In particular, a very interesting prediction [26] (see also [27]) was obtained for the large-j behavior of the anomalous dimension for twist-2 operators

$$\gamma(j) = a(z)\ln j, \quad z = \frac{\alpha_s N_c}{\pi} \tag{26}$$

in the strong coupling regime (see Ref. [28] for asymptotic corrections):

$$\lim_{z\to\infty} a = -z^{1/2} + \frac{3\ln 2}{8\pi} + \mathcal{O}\big(z^{-1/2}\big). \tag{27}$$

Here we took into account, that in our normalization $\gamma(j)$ contains the extra factor $-1/2$ in comparison with that in Ref. [26].

On the other hand, all anomalous dimensions $\gamma_i(j)$ and $\tilde{\gamma}_i(j)$ $(i=+,0,-)$ coincide at large-j and our results for $\gamma(j)$ allow one to find two first terms of the small-z expansion of the coefficient $a(z)$

$$\lim_{z\to 0} a = -z + \frac{\pi^2-1}{12}z^2 + \cdots. \tag{28}$$

To go from this expansion to the strong coupling regime we perform a resummation of the perturbative result using a method similar to the Pade approximation and taking into account, that for large N_c the perturbation series has a finite radius of convergency. Namely, we present the resumed coefficient $\tilde{a}$ as a solution of the simple algebraic equation

$$z = -\tilde{a} + \frac{\pi^2-1}{12}\tilde{a}^2. \tag{29}$$

From this equation the following large-z behaviour of $\tilde{a}$ is obtained:

$$\tilde{a} \approx -1.1632 z^{1/2} + 0.6765 + \mathcal{O}\big(z^{-1/2}\big) \tag{30}$$

in a rather good agreement with Eq. (27) based on the AdS/CFT correspondence. Note, that if we write for $\tilde{a}$ the more general equation

$$\sum_{k=1}^{n} B_k z^k = \sum_{r=1}^{2n} C_r \tilde{a}^r, \tag{31}$$

the coefficients B_k and C_r for $n \geqslant 2$ can be chosen in such way to include all known information about a.

Further, for $j \to 2$ due to the energy–momentum conservation

$$\gamma(j) = (j-2)\gamma'(2) + \cdots, \tag{32}$$

where the coefficient $\gamma'(2)$ can be calculated from our results in two first orders of the perturbation theory:

$$\gamma'(2) \simeq -1.6449z + 1.2158z^2. \tag{33}$$

Using the same method of resummation as we used above for $\tilde{a}$, we obtain for large-z:

$$\tilde{\gamma}'(2) \simeq -0.9071 z^{1/2} + 0.6768. \tag{34}$$

Let us take into account, that in this limit $\gamma = 1/2 + i\nu + (j-1)/2 \to 1 + (j-2)/2$ for the principal series of unitary representations of the Möbius group appearing in the BFKL equation [4]. Therefore we obtain for large-z:

$$j \simeq 2 - 1.1024 z^{-1/2} - 0.2148 z^{-1}, \tag{35}$$

in an agreement with the result, that the Pomeron in the strong coupling regime coincides with the graviton [29, 30]. The correction $\sim z^{-1/2}$ to the graviton spin $j = 2$ coincides in form with that obtained in Ref. [29] from the AdS/CFT correspondence and the coefficient in front of $z^{-1/2}$ was not calculated yet. Note, that for the soft Pomeron the correction is $\sim z^{-1}$ [30].

One can attempt to calculate the intercept of the BFKL Pomeron also using its perturbative expansion in Ref. [7]:

$$j - 1 = 2.7726 z - 5.0238 z^2. \tag{36}$$

After the Pade resummation we obtain in the strong coupling regime $j \simeq 2.5301 - 0.8444 z^{-1}$ in an reasonable agreement with the AdS/CFT estimate (see Ref. [30]). Note, however, that in the upper orders of the perturbation theory the BFKL equation should be modified by including the contributions from multi-gluon components of the Pomeron wave function.

In the conclusion we want to stress again, that the AdS/CFT correspondence unified with a resummation procedure gives a possibility to relate weak and strong coupling results.

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